



CLIFTON
COLLEGE

13+ Scholarship Examination
Specimen Paper

Mathematics

Time allowed: 90 minutes

- **Calculators are not allowed.**
- You may answer the questions in any order.
- If you get stuck on a question, move on to the next one.
- Write your answers separately, and show all of your working out clearly.

Question 1

Solve the following equations:

(i) $87 - 6a = 54$ (2)

(ii) $\frac{76+5b}{4} = 3b - 2$ (2)

(iii) $\frac{8c+9}{c+8} = c$ (2)

(iv) $\sqrt{5d - 6} = 7$ (2)

(8 marks for Question 1)

Question 2

(a) Amrita calculates that 40% of the number of her house is 48
What is the number of Amrita's house? (2)

(b) Ben works out that $\frac{7}{8}$ of the amount in his bank account is \$63
What is $\frac{8}{9}$ of the amount in Ben's bank account? (3)

(c) Carla and David each have a favourite number, c and d respectively.
Carla is delighted to find that $\frac{4}{5}$ of c is the same as 45% of d .

(i) Express the ratio $c : d$ in its simplest terms. (3)

(ii) Given that both c and d are two-digit positive integers and that their sum is a cube number, find c and d . (3)

(11 marks for Question 2)

Question 3

If $g = 4.5$ and $h = 0.6$ find the value of:

(i) gh (2)

(ii) $\frac{g}{h}$ (2)

(iii) $\sqrt{\frac{g+h+3}{10}}$ (2)

(6 marks for Question 3)

Question 4

(a) Solve the equation $a + 2(3a - 4) + 5 = 6(a + 7) - 8a$ (3)

(b) (i) Solve each of the following inequalities:

(a) $5x - 4 \leq 3(2x - 1)$
(2)

(b) $6 - 7x > -8$ (2)

(ii) List the whole numbers which satisfy both these inequalities (1)

(8 marks for Question 4)

Question 5

Isobel has been trying to work out the ages of her neighbours, Judith and Kate.

Judith told Isobel:

‘Twice my age is equal to three times Kate’s age.’

Let j and k be the ages of Judith and Kate respectively.

(i) Write down an equation in j and k . (2)

(ii) Write down expressions for

(a) Judith’s age two years ago (1)

(b) Kate’s age in three years’ time (1)

Kate told Isobel:

‘Three times what Judith’s age was two years ago is equal to four times what my age will be in three years’ time.’

(iii) Show that $3j - 4k = 18$ (2)

(iv) Solve the equations in parts (i) and (iii). (3)

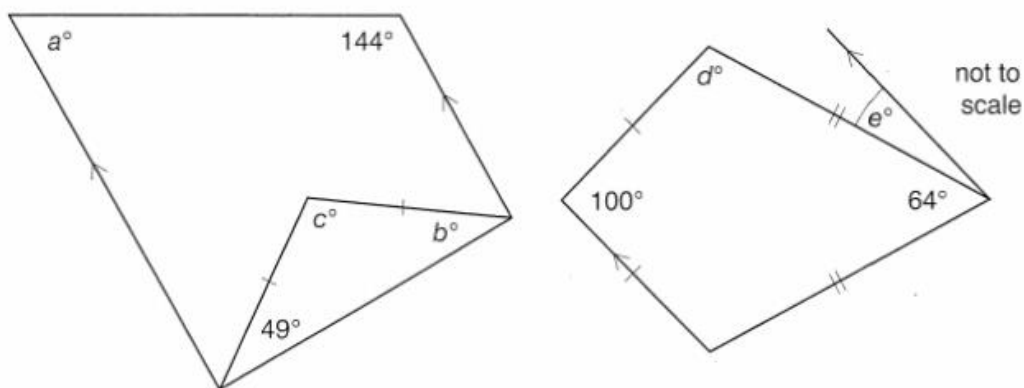
Kate’s age is now two-thirds of Judith’s age.

(v) How many years ago was Kate’s age half of Judith’s age? (2)

(11 marks for Question 5)

Question 6

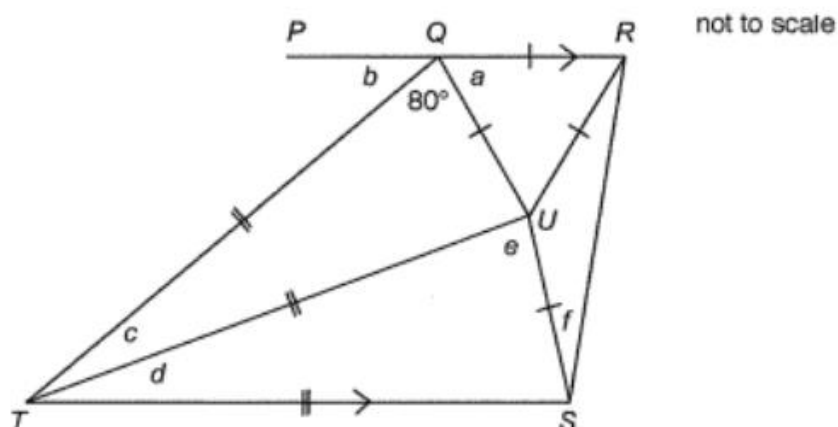
Find the value of each of the angles a , b , c , d and e marked in the following diagrams:



(1, 1, 2, 2, 2)

(8 marks for Question 6)

Question 7



In the diagram $QRST$ is a trapezium,

QRU is an equilateral triangle

And TUQ , TUS and SUR are all isosceles triangles.

PQR is a straight line and angle $TQU = 80^\circ$.

Find the size of each of the angles a , b , c , d , e and f .

(1, 1, 1, 2, 2, 2)

(9 marks for Question 7)

Question 8

You may have come across the following sequence of *Fibonacci* numbers before:

1 1 2 3 5 8 13 21 34 55 89 144

After the first two terms (1 and 1), each term is the sum of the previous two terms.

For example:

$$1 + 1 = 2, \quad 5 + 8 = 13, \quad 34 + 55 = 89$$

- (i) Write down the next two *Fibonacci* numbers which come after 144 (1)

We shall now explore some of the many interesting patterns with *Fibonacci* numbers.

For any four consecutive *Fibonacci* numbers, the product of the first and fourth is either one more or one less than the product of the second and third (depending on whether the first number is an odd- or even-numbered term in the sequence).

For example: $3 \times 13 = 39$ is one less than $5 \times 8 = 40$

but

$$34 \times 144 = 4896 \text{ is one more than } 55 \times 89 = 4895$$

- (ii) (a) Given that $21 \times 89 = 1869$, find 34×55 (1)

- (b) Find $8 \times 34 - 13 \times 21 + 13 \times 55 - 21 \times 34$ (2)

For any **ten** consecutive *Fibonacci* numbers, their sum is 11 times the seventh of the numbers, for example:

$$2 + 3 + 5 + 8 + 13 + 21 + 34 + 55 + 89 + 144 = 374 = 11 \times 34$$

- (iii) The sum of a set of 10 consecutive *Fibonacci* numbers is 2563
What is the **first** term of this set? (2)

The sum of the **squares** of the *Fibonacci* numbers from 1, 1, 2, 3, 5, etc. is the product of the last number with the *Fibonacci* number which follows it.

For example:

$$\begin{aligned} 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 &= 1 + 1 + 4 + 9 + 25 + 64 + 169 + 441 \\ &= 714 \\ &= 21 \times 34 \end{aligned}$$

- (iv) Find $5^2 + 8^2 + 13^2 + 21^2 + 34^2 + 55^2$ (2)

(8 Marks for Question 8)

Question 9

- (i) Express 252 as the product of its prime factors using indices. (2)

Caroline wrote down a list of all the factors of 252, but some of them were illegible, and replaced with the letters p, q, r and s in the list below.

1, 2, 3, 4, 6, 7, p , 12, q , 18, 21, 28, 36, r , 63, s , 126, 252

- (ii) Find all the missing factors p, q, r and s . (3)

Caroline also wrote 525 as the product of its prime factors:

$$525 = 3 \times 5^2 \times 7$$

She also calculated that $252 \times 525 = 132300$

- (iii) Express 132300 as the product of its prime factors

Caroline noticed that 44100 is the largest square number which is a factor of 132300 (2)

- (iv) Find the square root of 44100 (2)

(9 Marks for Question 9)

Question 10

At the end of a market day there was one large box of apples left unsold, so the farmer decided to give the apples to his horses.

The farmer worked out that if half the horses had 7 apples each and the other half had 6 apples each, there would be only 2 apples left.

For this question, you should let a and h be the number of apples and horses respectively.

- (i) Form an equation, in terms of a and h , and show that it simplifies to

$$2a = 13h + 4 \quad (3)$$

The farmer's wife pointed out that if he gave 7 apples each to 7 horses and 6 apples each to the rest of the horses, there would be 1 apple left for her.

- (ii) Form another equation in terms of a and h . (3)

- (iii) Solve your equation from parts (i) and (ii) to find a and h . (4)

The farmer then noticed that a quarter of the apples were inedible.

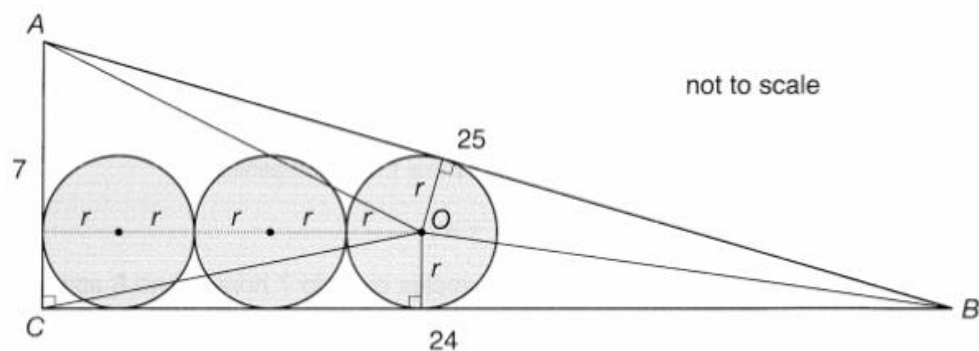
- (iv) If the rest were shared equally, how many apples would each horse get? (1)

(11 Marks for Question 10)

Question 11

A right-angle triangle ABC is shown below.

The length of each side is shown in centimetres.



- (i) Find the area of triangle ABC . (2)

Inside triangle ABC are three identical circles.

The circles at the left and right ends each touch the middle circle and two sides of the triangle.

The middle circle touches side BC .

Each circle has radius r cm.

- (ii) Show that the area of triangle AOC is $17\frac{1}{2}r$ cm². (1)
- (iii) Find similar expressions for
- (a) the area of triangle BOC (1)
- (b) the area of triangle AOB (1)
- (iv) Use your answers to parts (ii) and (iii) to write an expression, in terms of r , for the total area of triangle ABC . (2)
- (v) Use your answers to parts (i) and (iv) to write an equation and solve it to find the value of r . (1)
- (vi) By making a suitable approximation for π , **estimate** the percentage of the triangle which is shaded within the three circles, correct to the nearest 5%. (3)

(11 Marks for Question 11)

END OF PAPER
100 marks in total